

# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools



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The goal of this practice guide is to formulate specific and coherent evidence-based recommendations for use by educators addressing the challenge of reducing the number of children who struggle with mathematics by using “response to intervention” (RtI) as a means of both identifying students who need more help and providing these students with high-quality interventions. The guide provides practical, clear information on critical topics related to RtI and is based on the best available evidence as judged by the panel. Recommendations in this guide should not be construed to imply that no further research is warranted on the effectiveness of particular strategies used in RtI for students struggling with mathematics.

# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

**April 2009**

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# Introduction

Students struggling with mathematics may benefit from early interventions aimed at improving their mathematics ability and ultimately preventing subsequent failure. This guide provides eight specific recommendations intended to help teachers, principals, and school administrators use Response to Intervention (RtI) to identify students who need assistance in mathematics and to address the needs of these students through focused interventions. The guide provides suggestions on how to carry out each recommendation and explains how educators can overcome potential roadblocks to implementing the recommendations.

The recommendations were developed by a panel of researchers and practitioners with expertise in various dimensions of this topic. The panel includes a research mathematician active in issues related to K–8 mathematics education, two professors of mathematics education, several special educators, and a mathematics coach currently providing professional development in mathematics in schools. The panel members worked collaboratively to develop recommendations based on the best available research evidence and our expertise in mathematics, special education, research, and practice.

The body of evidence we considered in developing these recommendations included evaluations of mathematics interventions for low-performing students and students with learning disabilities. The panel considered high-quality experimental and quasi-experimental studies, such as those meeting the criteria of the What Works Clearinghouse (<http://www.whatworks.ed.gov>), to provide the strongest evidence of effectiveness. We also examined studies of the technical adequacy of batteries of screening and progress monitoring measures for recommendations relating to assessment.

In some cases, recommendations reflect evidence-based practices that have been demonstrated as effective through rigorous research. In other cases, when such evidence is not available, the recommendations reflect what this panel believes are best practices. Throughout the guide, we clearly indicate the quality of the evidence that supports each recommendation.

Each recommendation receives a rating based on the strength of the research evidence that has shown the effectiveness of a recommendation (table 1). These ratings—*strong*, *moderate*, or *low*—have been defined as follows:

*Strong* refers to consistent and generalizable evidence that an intervention program causes better outcomes.<sup>1</sup>

*Moderate* refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated)—or to evidence from studies that are generalizable but have more causal ambiguity than offered by experimental designs (such as statistical models of correlational data or group comparison designs for which the equivalence of the groups at pretest is uncertain).

*Low* refers to expert opinion based on reasonable extrapolations from research and theory on other topics and evidence from studies that do not meet the standards for moderate or strong evidence.

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1. Following WWC guidelines, we consider a positive, statistically significant effect or large effect size (i.e., greater than 0.25) as an indicator of positive effects.

**Table 1. Institute of Education Sciences levels of evidence for practice guides**

<b>Strong</b>	<p>In general, characterization of the evidence for a recommendation as strong requires both studies with high internal validity (i.e., studies whose designs can support causal conclusions) and studies with high external validity (i.e., studies that in total include enough of the range of participants and settings on which the recommendation is focused to support the conclusion that the results can be generalized to those participants and settings). Strong evidence for this practice guide is operationalized as:</p> <ul style="list-style-type: none"> <li>• A systematic review of research that generally meets the standards of the What Works Clearinghouse (WWC) (see <a href="http://ies.ed.gov/ncee/wwc/">http://ies.ed.gov/ncee/wwc/</a>) and supports the effectiveness of a program, practice, or approach with no contradictory evidence of similar quality; OR</li> <li>• Several well-designed, randomized controlled trials or well-designed quasi-experiments that generally meet the standards of WWC and support the effectiveness of a program, practice, or approach, with no contradictory evidence of similar quality; OR</li> <li>• One large, well-designed, randomized controlled, multisite trial that meets WWC standards and supports the effectiveness of a program, practice, or approach, with no contradictory evidence of similar quality; OR</li> <li>• For assessments, evidence of reliability and validity that meets the Standards for Educational and Psychological Testing.<sup>a</sup></li> </ul>
<b>Moderate</b>	<p>In general, characterization of the evidence for a recommendation as moderate requires studies with high internal validity but moderate external validity, or studies with high external validity but moderate internal validity. In other words, moderate evidence is derived from studies that support strong causal conclusions but when generalization is uncertain, or studies that support the generality of a relationship but when the causality is uncertain. Moderate evidence for this practice guide is operationalized as:</p> <ul style="list-style-type: none"> <li>• Experiments or quasi-experiments generally meeting the standards of WWC and supporting the effectiveness of a program, practice, or approach with small sample sizes and/or other conditions of implementation or analysis that limit generalizability and no contrary evidence; OR</li> <li>• Comparison group studies that do not demonstrate equivalence of groups at pretest and therefore do not meet the standards of WWC but that (a) consistently show enhanced outcomes for participants experiencing a particular program, practice, or approach and (b) have no major flaws related to internal validity other than lack of demonstrated equivalence at pretest (e.g., only one teacher or one class per condition, unequal amounts of instructional time, highly biased outcome measures); OR</li> <li>• Correlational research with strong statistical controls for selection bias and for discerning influence of endogenous factors and no contrary evidence; OR</li> <li>• For assessments, evidence of reliability that meets the Standards for Educational and Psychological Testing<sup>b</sup> but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.</li> </ul>
<b>Low</b>	<p>In general, characterization of the evidence for a recommendation as low means that the recommendation is based on expert opinion derived from strong findings or theories in related areas and/or expert opinion buttressed by direct evidence that does not rise to the moderate or strong levels. Low evidence is operationalized as evidence not meeting the standards for the moderate or high levels.</p>

a. American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (1999).

b. Ibid.



## The What Works Clearinghouse standards and their relevance to this guide

The panel relied on WWC evidence standards to assess the quality of evidence supporting mathematics intervention programs and practices. The WWC addresses evidence for the causal validity of instructional programs and practices according to WWC standards. Information about these standards is available at <http://ies.ed.gov/ncee/wwc/references/standards/>. The technical quality of each study is rated and placed into one of three categories:

- *Meets Evidence Standards*—for randomized controlled trials and regression discontinuity studies that provide the strongest evidence of causal validity.
- *Meets Evidence Standards with Reservations*—for all quasi-experimental studies with no design flaws and randomized controlled trials that have problems with randomization, attrition, or disruption.
- *Does Not Meet Evidence Screens*—for studies that do not provide strong evidence of causal validity.

Following the recommendations and suggestions for carrying out the recommendations, Appendix D presents information on the research evidence to support the recommendations.

The panel would like to thank Kelly Haymond for her contributions to the analysis, the WWC reviewers for their contribution to the project, and Jo Ellen Kerr and Jamila Henderson for their support of the intricate logistics of the project. We also would like to thank Scott Cody for his oversight of the overall progress of the practice guide.

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# Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools

## Overview

Response to Intervention (RtI) is an early detection, prevention, and support system that identifies struggling students and assists them before they fall behind. In the 2004 reauthorization of the Individuals with Disabilities Education Act (PL 108-446), states were encouraged to use RtI to accurately identify students with learning disabilities and encouraged to provide additional supports for students with academic difficulties regardless of disability classification. Although many states have already begun to implement RtI in the area of reading, RtI initiatives for mathematics are relatively new.

Students' low achievement in mathematics is a matter of national concern. The recent National Mathematics Advisory Panel Report released in 2008 summarized the poor showing of students in the United States on international comparisons of mathematics performance such as the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA).<sup>2</sup> A recent survey of algebra teachers associated with the report identified key deficiencies of students entering algebra, including aspects of whole number arithmetic, fractions, ratios, and proportions.<sup>3</sup> The National Mathematics Advisory Panel

concluded that all students should receive preparation from an early age to ensure their later success in algebra. In particular, the report emphasized the need for mathematics interventions that mitigate and prevent mathematics difficulties.

This panel believes that schools can use an RtI framework to help struggling students prepare for later success in mathematics. To date, little research has been conducted to identify the most effective ways to initiate and implement RtI frameworks for mathematics. However, there is a rich body of research on effective mathematics interventions implemented outside an RtI framework. Our goal in this practice guide is to provide suggestions for assessing students' mathematics abilities and implementing mathematics interventions within an RtI framework, in a way that reflects the best evidence on effective practices in mathematics interventions.

RtI begins with high-quality instruction and universal screening for all students. Whereas high-quality instruction seeks to prevent mathematics difficulties, screening allows for early detection of difficulties if they emerge. Intensive interventions are then provided to support students in need of assistance with mathematics learning.<sup>4</sup> Student responses to intervention are measured to determine whether they have made adequate progress and (1) no longer need intervention, (2) continue to need some intervention, or (3) need more intensive intervention. The levels of intervention are conventionally referred to as "tiers." RtI is typically thought of as having three tiers.<sup>5</sup> Within a three-tiered RtI model, each tier is defined by specific characteristics.

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2. See, for example, National Mathematics Advisory Panel (2008) and Schmidt and Houang (2007). For more information on the TIMSS, see <http://nces.ed.gov/timss/>. For more information on PISA, see <http://www.oecd.org>.

3. National Mathematics Advisory Panel (2008).

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4. Fuchs, Fuchs, Craddock et al. (2008).

5. Fuchs, Fuchs, and Vaughn (2008) make the case for a three-tier RtI model. Note, however, that some states and school districts have implemented multitier intervention systems with more than three tiers.

- Tier 1 is the mathematics instruction that all students in a classroom receive. It entails universal screening of all students, regardless of mathematics proficiency, using valid measures to identify students at risk for future academic failure—so that they can receive early intervention.<sup>6</sup> There is no clear consensus on the characteristics of instruction other than that it is “high quality.”<sup>7</sup>
- In tier 2 interventions, schools provide additional assistance to students who demonstrate difficulties on screening measures or who demonstrate weak progress.<sup>8</sup> Tier 2 students receive supplemental small group mathematics instruction aimed at building targeted mathematics proficiencies.<sup>9</sup> These interventions are typically provided for 20 to 40 minutes, four to five times each week.<sup>10</sup> Student progress is monitored throughout the intervention.<sup>11</sup>
- Tier 3 interventions are provided to students who are not benefiting from tier 2 and require more intensive assistance.<sup>12</sup> Tier 3 usually entails one-on-one tutoring along with an appropriate mix of instructional interventions. In some cases, special education services are included in tier 3, and in others special education is considered an additional tier.<sup>13</sup> Ongoing analysis of

student performance data is critical in this tier. Typically, specialized personnel, such as special education teachers and school psychologists, are involved in tier 3 and special education services.<sup>14</sup> However, students often receive relevant mathematics interventions from a wide array of school personnel, including their classroom teacher.

## Summary of the Recommendations

This practice guide offers eight recommendations for identifying and supporting students struggling in mathematics (table 2). The recommendations are intended to be implemented within an RtI framework (typically three-tiered). The panel chose to limit its discussion of tier 1 to universal screening practices (i.e., the guide does not make recommendations for general classroom mathematics instruction). Recommendation 1 provides specific suggestions for conducting universal screening effectively. For RtI tiers 2 and 3, recommendations 2 through 8 focus on the most effective content and pedagogical practices that can be included in mathematics interventions.

Throughout this guide, we use the term “interventionist” to refer to those teaching the intervention. At a given school, the interventionist may be the general classroom teacher, a mathematics coach, a special education instructor, other certified school personnel, or an instructional assistant. The panel recognizes that schools rely on different personnel to fill these roles depending on state policy, school resources, and preferences.

Recommendation 1 addresses the type of screening measures that should be used in tier 1. We note that there is more research on valid screening measures for students in

6. For reviews see Jiban and Deno (2007); Fuchs, Fuchs, Compton et al. (2007); Gersten, Jordan, and Flojo (2005).

7. National Mathematics Advisory Panel (2008); National Research Council (2001).

8. Fuchs, Fuchs, Craddock et al. (2008); National Joint Committee on Learning Disabilities (2005).

9. Fuchs, Fuchs, Craddock et al. (2008).

10. For example, see Jitendra et al. (1998) and Fuchs, Fuchs, Craddock et al. (2008).

11. National Joint Committee on Learning Disabilities (2005).

12. Fuchs, Fuchs, Craddock et al. (2008).

13. Fuchs, Fuchs, Craddock et al. (2008); National Joint Committee on Learning Disabilities (2005).

14. National Joint Committee on Learning Disabilities (2005).

**Table 2. Recommendations and corresponding levels of evidence**

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	<b>Moderate</b>
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	<b>Low</b>
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	<b>Strong</b>
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	<b>Strong</b>
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	<b>Moderate</b>
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	<b>Moderate</b>
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	<b>Low</b>
8. Include motivational strategies in tier 2 and tier 3 interventions.	<b>Low</b>

Source: Authors' compilation based on analysis described in text.